EX. 2.6 FREE PTC

$$\Psi(x,0) = \begin{cases} 1/\sqrt{2a} & (-a < x < a) \\ 0 & \text{ELSEWHERE} \end{cases}$$

$$\Psi(x,t) = \frac{1}{\pi\sqrt{2a}} \int_{-\infty}^{+\infty} \frac{\sin(ka)}{k} e^{i\left(kx - \frac{\hbar k^2}{2m}t\right)} dk$$

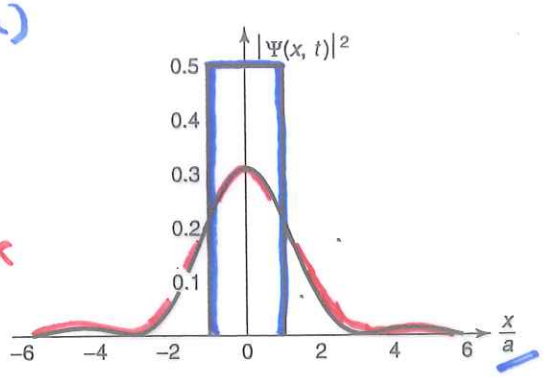

FIGURE 2.8: Graph of $|\Psi(x,t)|^2$ (Equation 2.104) at $t = 0$ (the rectangle) and at $t = ma^2/\hbar$ (the curve).

LIMITING CASES:

* VERY SMALL a

$\Psi(x,0)$ LIKE A SPIKE $\sin(ka) \approx ka$

$$\phi(k) \simeq \frac{1}{\sqrt{\pi a}} \frac{ka}{k} = \sqrt{\frac{a}{\pi}}$$

CONSTANT

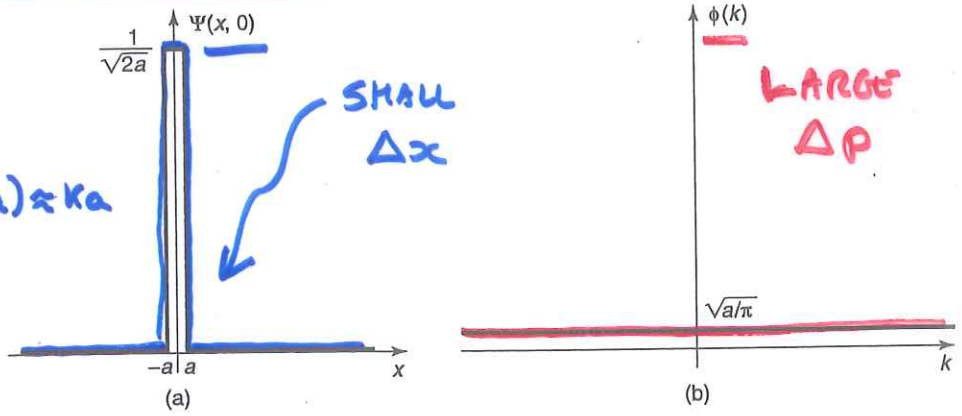

FIGURE 2.9: Example 2.6, for small a. (a) Graph of $\Psi(x,0)$. (b) Graph of $\phi(k)$.

* VERY LARGE a

$$\phi(k) = \sqrt{\frac{a}{\pi}} \frac{\sin(ka)}{ka}$$

$$\psi(x,0) = \frac{1}{\sqrt{2a}} \quad \text{const}$$

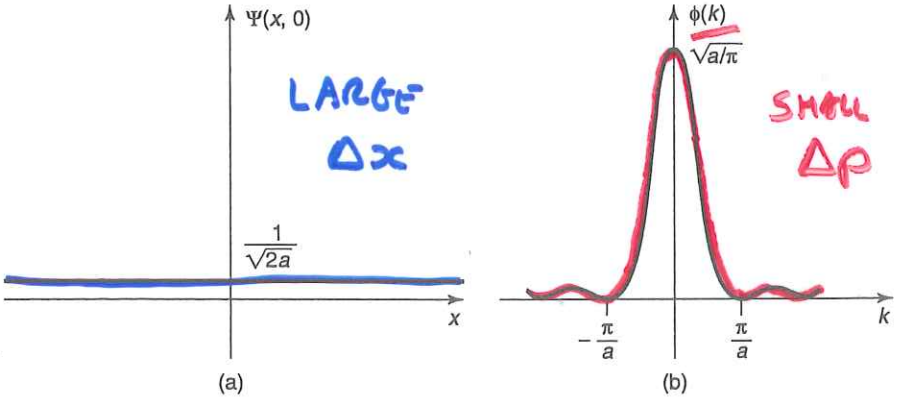

FIGURE 2.10: Example 2.6, for large a. (a) Graph of $\Psi(x,0)$. (b) Graph of $\phi(k)$.

$$v_{phase} = \frac{\omega}{k} = \frac{\hbar k}{2m} = \frac{1}{2} v_{class.}$$

$$v_{group} = \frac{d\omega}{dk} = \frac{\hbar k}{m} = v_{class}$$

CORRECT!

v_g GROUP VELOCITY

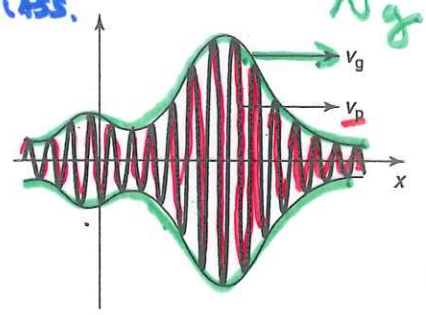

FIGURE 2.11: A wave packet. The "envelope" travels at the group velocity; the "ripples" travel at the phase velocity.