Newtonian Fractional-Dimension Gravity and the Mass-Dimension Field Equation

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Introduction

Newtonian Fractional-Dimension Gravity (NFDG) is an alternative model of gravity, which was introduced in 2020 with the first three papers on the subject. Its goal is to model galaxy rotation curves (RCs) without using the controversial Dark Matter (DM). This is done by assuming that galactic structures behave as fractal media, with an effective dimension which can be lower than the standard value D = 3, including possible fractional, i.e., non-integer dimensions. This model is broadly based on the methods of fractional calculus and is connected with Modified Newtonian Dynamics (MOND). A fourth paper (2021) introduced a relativistic extension (RFDG), which might be useful for cosmological applications. A fifth paper (2022) analyzed additional galaxies in relation to the External Field Effect (EFE). A sixth paper (2023), considered some cases of galaxies with little or no dark matter and the Bullet Cluster merger. A recent paper (2025) examined additional galaxies and introduced a fundamental *Mass-Dimension Field Equation.*

A brief review of NFDG

NFDG is based on the mathematical theory of spaces with non-integer dimension applied to the Newtonian law of gravitation for a space-time D+1, where $D \leq 3$ is a non-integer space dimension. The NFDG gravitational potential for a point-like mass m is:

$$\Phi_{NFDG}(r) = -\frac{2\pi^{1-D/2}\,\Gamma(D/2)\,Gm}{(D-2)\,l_0 r^{D-2}}\;;\; D \neq 2$$

$$\Phi_{NFDG}(r) = \frac{2\,Gm}{l_0}\ln r\;;\; D = 2,$$

where G is the gravitational constant, r is a dimensionless radial coordinate, and l_0 is an appropriate scale length. This potential can be generalized to the spherical bulge and cylindrical disk stellar/gas mass distributions of any galaxy. The dimension function D=D(R) can be obtained directly from the experimental RCs data, or derived from a more foundational field equation, as a *mass dimension function* $D=D_m(R)$:

$$D_m(R) - 1 + R\ln\left(\frac{R}{R_0}\right)\frac{dD_m(R)}{dR} = \frac{2\pi^{\left(\frac{D_m(R)-1}{2}\right)}}{\Gamma\left(\frac{D_m(R)-1}{2}\right)}\,\frac{\widetilde{\Sigma}_{tot}(R/R_0)}{M_0}.$$

The NFDG gravitational field follows from the generalized potentials, supplemented by the dimension function: $\mathbf{g}_{NFDG}(R) = -\frac{1}{l_0}\frac{d\Phi_{NFDG}(R)}{dR}\hat{\mathbf{R}}.$

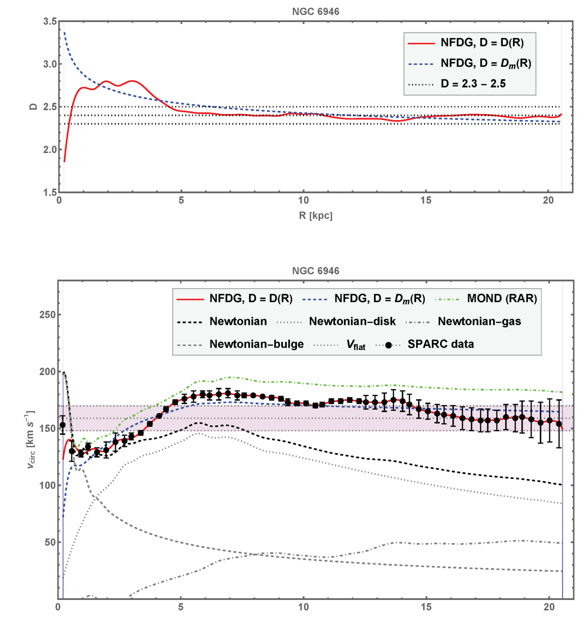
The circular velocity, for stars rotating in the main galactic plane, also follows from the above equations:

$$v_{circ}(R) = \sqrt{l_0 R\,|\mathbf{g}_{NFDG}(R)|},$$

with the NFDG field computed by using either the standard D=D(R), or the new *mass-dimension function* $D=D_m(R)$.

NGC 6946

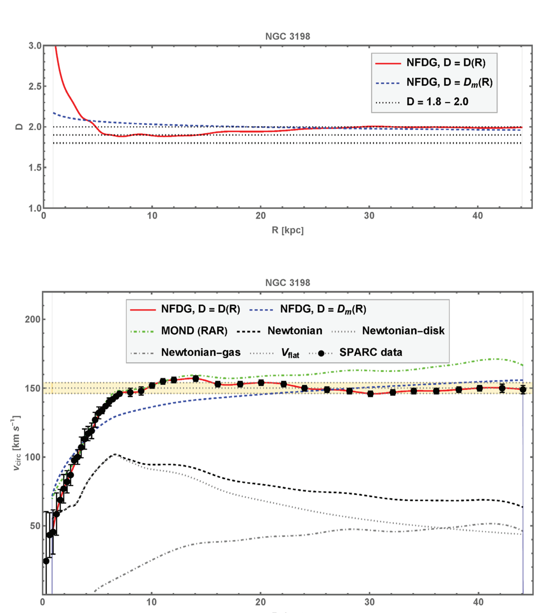
NFDG analysis of this bulge-dominated galaxy: top panel shows the variable dimensions D(R) (red-solid) and $D=D_m(R)$ (blue-dashed), computed with NFDG models. Bottom panel: NFDG RCs compared with SPARC data and MOND (RAR) predictions. A perfect fit without any DM is obtained by using our main D(R) function (red-solid curve). A slighly less effective fit is obtained by using the *mass-dimension function* $D=D_m(R)$ (blue-dashed).



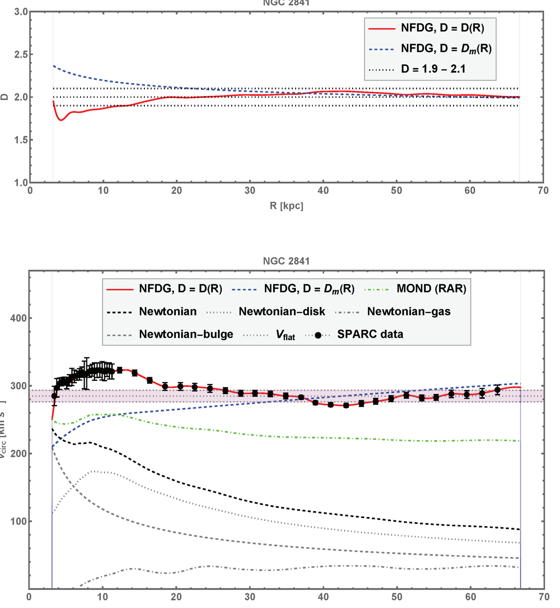
NGC 6946 (A. Gusan)

NGC 3198 & NGC 2841

NFDG analysis of disk-dominated barred spiral galaxy NGC 3198 and unbarred spiral galaxy NGC 2841. Top panels show the NFDG variable dimensions, while the bottom panels illustrate the NFDG velocity curves.



NGC 3198 (M.T. Patterson)

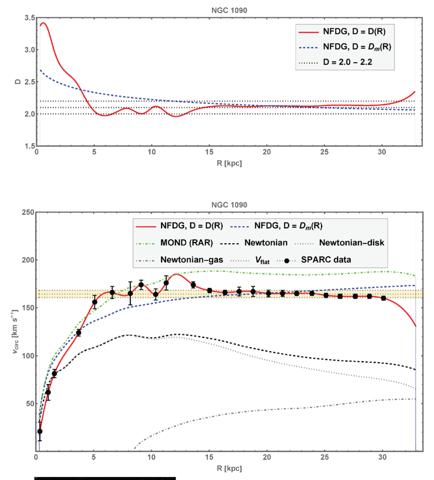


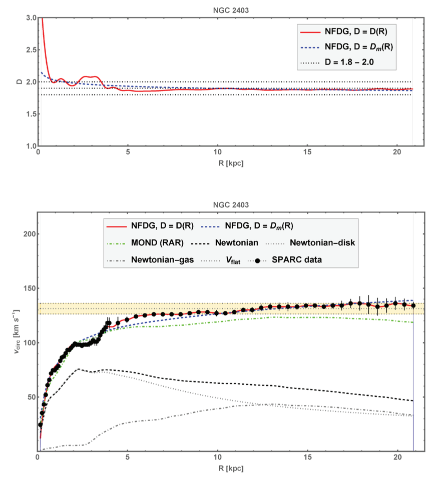
NGC 2841 (NASA/ESA)

Additional NFDG results

NGC 1090, NGC 2403, NGC 5055, and NGC 6503

Four additional disk-dominated galaxies were studied with NFDG methods, in relation to the mass-dimension field equation and the corresponding function $D=D_m(R)$. These four galaxies were modeled like the previous ones and very effective fits to their galactic rotation data were produced, as shown below.

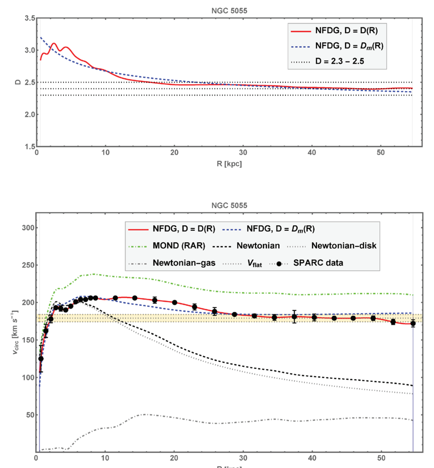


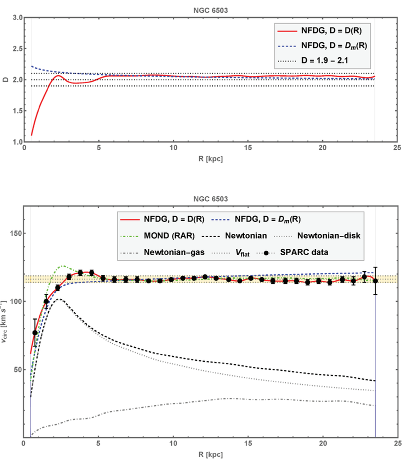


NGC 1090 (SDSS)

NGC 2403 (Subaru-HST)

Disk-dominated galaxies are characterized in NFDG by fractional dimension values close to D=2. This is due to the axially-symmetric, flattened disk shapes of these galaxies. As a result, their fractal structures behave more like a 2-dimensional object than a 3-dimensional one. The gas and bulge components are less important for these galaxies.



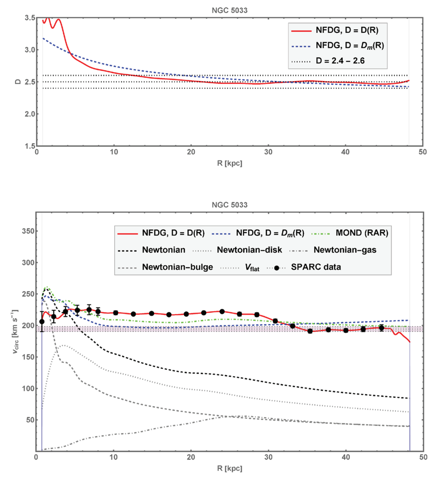


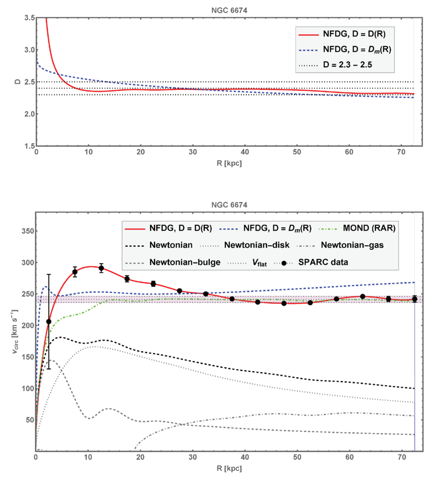
NGC 5055 (NASA/ESA)

NGC 6503 (NASA/ESA)

NGC 5033, NGC 6674, NGC 7814, and NGC 3741

Three bulge-dominated galaxies (NGC 5033, 6674, 7814) and one gas-dominated galaxy (NGC 3741) were also studied with NFDG methods. These additional galaxies were modeled as in the previous cases and high-quality galactic rotation curves were obtained, as seen in the figures below.

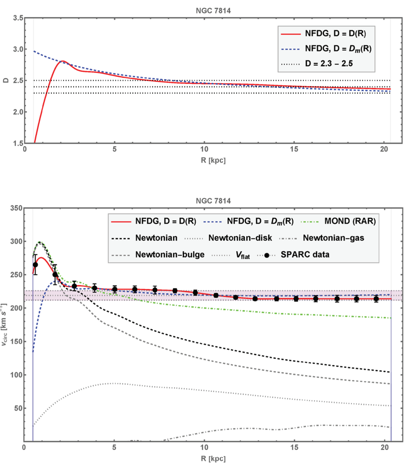


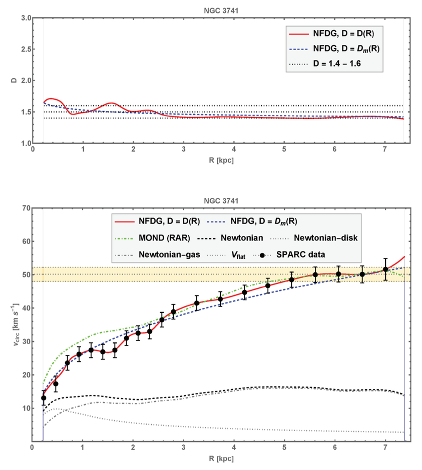


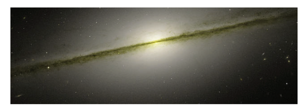
NGC 5033 (U. Arizona)

NGC 6674 (K. Biggs)

Bulge-dominated galaxies are characterized in NFDG by fractional dimensions between D=2 and D=3. This is due to the spherical symmetry of the dominating bulge component. On the contrary, gas-dominated galaxies show fractional dimensions between D=1 and D=2. The prominent gas component lowers considerably their fractal dimension.







NGC 7814 (NASA/ESA)

NGC 3741 (NASA/ESA)

Conclusions

In this work, Newtonian gravity was extended to spaces with non-integer dimension D. The NFDG model can be used to fit galaxy rotation curves without any dark matter components, assuming that each galaxy is described by a characteristic variable dimension function D(R), or $D_m(R)$, derived from the new mass-dimension field equation (published paper: *Varieschi, G.U. Newtonian Fractional-Dimension Gravity and the Mass-Dimension Field Equation. Universe 2025, 11, 388*).
For more details, see the NFDG webpage at: https://gvarieschi.lmu.build/NFDG2020.html and links to all NFDG papers therein.